PART 1

INTRODUCTION

Part 1 begins your introduction to chemical engineering calculations by reviewing certain topics underlying the main principles to be discussed. You have already encountered most of these concepts in your basic chemistry and physics courses. Why, then, the need for a review? First, from experience we have found it necessary to restate these familiar basic concepts in a somewhat more precise and clearer fashion; second, you will need practice to develop your ability to analyze and work engineering problems. If you encounter new material as you go through these chapters, or if you flounder over little gaps in your skills or knowledge of old material, you should devote extra attention to the chapters by solving extra problems in the set that you will find at the end of each chapter. To read and understand the principles discussed in these chapters is relatively easy; to apply them to different unfamiliar situations is not. An engineer becomes competent in his or her profession by mastering the techniques developed by one's predecessors—thereafter comes the time to pioneer new ones.

What I hear, I forget;
What I see, I remember;
What I do, I understand.

Confucius

Part 1 begins with a discussion of units, dimensions, and conversion factors, and then goes on to review some terms you should already be acquainted with, including:

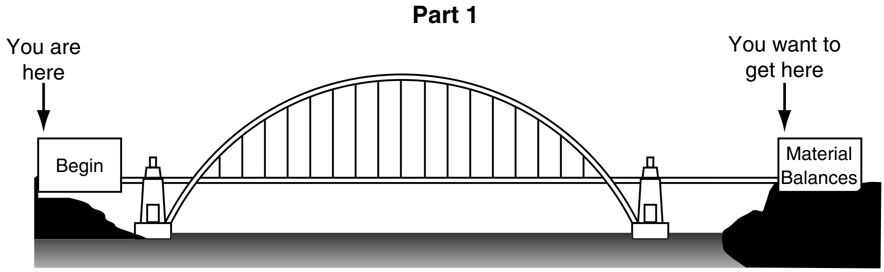

Figure Part 1.1 The bridge to success.

a. Mole and mole fraction
b. Density and specific gravity
c. Measures of concentration
d. Temperature
e. Pressure

A firm grasp of this information as presented in the next five chapters will help guarantee "plug-and-play" acquisition of the information in the remaining chapters. You will find that adding new ideas and techniques will be as easy as copying and pasting images in a computer. Consider the following story.

> One night a group of nomads were preparing to retire for the evening when suddenly they were surrounded by a great light. They knew they were in the presence of a celestial being. With great anticipation, they awaited a heavenly message of great importance that they knew must be especially for them.
>
> Finally, the voice spoke.
>
> "Gather as many pebbles as you can. Put them in your saddle bags. Travel a day's journey and tomorrow night will find you glad and it will find you sad."
>
> After the light departed, the nomads shared their disappointment and anger with each other. They had expected the revelation of a great universal truth that would enable them to create wealth, health, and purpose for the world. But instead they were given a menial task that made no sense to them at all. However, the memory of the brilliance of their visitor caused each one to pick up a few pebbles and deposit them in their saddle bags while voicing their displeasure.
>
> They traveled a day's journey and that night while making camp, they reached into their saddle bags and discovered every pebble they had gathered had become a diamond. They were glad they had diamonds. They were sad they had not gathered more pebbles.*

*Schlatter, J. W., quoted in *A Second Helping of Chicken Soup for the Soul,* J. Canfield and M. Hansen (eds.), Health Communications, Deerfield Beach, FL (1995).

SUPPLEMENTARY REFERENCES

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CHAPTER 1

DIMENSIONS, UNITS, AND THEIR CONVERSION

Your objectives in studying this chapter are to be able to:

1. Understand and explain the difference between dimensions and units.
2. Add, subtract, multiply, and divide units associated with numbers.
3. Specify the basic and derived units in the SI and American Engineering (AE) systems for mass, length, volume, density, and time, and their equivalents.
4. Convert one set of units in a function or equation into another equivalent set for mass, length, area, volume, time, and force.
5. Explain the difference between weight and mass.
6. Define and know when to use the gravitational conversion factor g_c.
7. Apply the concepts of dimensional consistency to determine the validity of an equation or function.
8. Employ an appropriate number of significant figures in your calculations.

"Take care of your units and they will take care of you."

Anonymous

At some time in every engineer's life comes the exasperating sensation of frustration in problem solving. Somehow, the answers or the calculations do not come out as expected. Often this outcome arises because of errors in the handling of units.

The use of units along with the numbers in your calculations requires more attention than you probably have been giving to your computations in the past. In addition, you will discover that checking the consistency of units in your equations will prove to be a valuable tool that will reduce the number of errors you commit when performing engineering calculations.

Looking Ahead

In this chapter we review the SI and American Engineering systems of units, show how conversions between units can be accomplished efficiently, and discuss the concept of dimensional homogeneity (consistency). We also provide some comments with respect to the number of significant figures to use in your calculations.

1.1 Units and Dimensions

Engineers and scientists have to be able to communicate not only with words but also by carefully defined numerical descriptions. Read the following news report that appeared in the *Wall Street Journal*, June 6, 2001, on page A22:

> SEOUL, South Korea—A mix up in the cockpit over whether altitude guidance was measured in feet or meters led to the crash of a Korean Air Lines McDonnell Douglas MD-11 freighter soon after takeoff in Shanghai in April 1999, investigators said.
>
> The crash killed all three crew-members. Five people on the ground were killed and 40 more were injured when the plane went down in light rain onto a construction site near Shanghai's Hongqiao Airport.
>
> According to a summary of the crash report released by South Korean authorities, a Chinese air-traffic controller directed the pilots to an altitude of 1,500 meters (4,950 feet). The plane was climbing rapidly to that level when the copilot told the pilot he thought the instructed height was 1,500 feet, equivalent to 455 meters. The international aviation industry commonly measures altitude in feet, and the confusion led the pilot to conclude the jet was almost 1,000 meters too high, so he quickly moved the controls to lower the plane. As the plane descended, the pilot realized the error but couldn't correct the mistake in time.
>
> South Korea's Ministry of Construction and Transportation said Korean Air Lines would lose the right to serve the Seoul-Shanghai cargo route for at least two years because of errors by the pilots. Korean Air Lines said it would appeal the decision . . .

Now you can understand the point of defining your quantities carefully so that your communications are understood.

1.1-1 What Are Units and Dimensions and How Do They Differ?

Dimensions are our basic concepts of measurement such as *length, time, mass, temperature*, and so on; **units** are the means of expressing the dimensions, such as *feet* or *centimeters* for length, and *hours* or *seconds* for time. By attaching units to all numbers that are not fundamentally dimensionless, you get the following very practical benefits:

a. diminished possibility of errors in your calculations,
b. reduced intermediate calculations and time in problem solving,
c. a logical approach to the problem rather than remembering a formula and substituting numbers into the formula,
d. easy interpretation of the physical meaning of the numbers you use.

In this book you will use the two most commonly used systems of units:

1. **SI**, formally called Le Systeme Internationale d'Unites, and informally called SI or more often (redundantly) the SI system of units.
2. **AE**, or American Engineering system of units, not to be confused with what is called the U.S. Conventional System (USCS) nor the English system of units.

The SI system has certain advantages over the AE system in that fewer names are associated with the dimensions, and conversion of one set of units to another is easier, but in the United States the AE system has deep roots. Most modern computer programs (e.g., process simulators) allow the use of either or mixed sets of units.

Dimensions and their respective units are classified as fundamental or derived:

- **Fundamental** (or basic) dimensions/units are those that can be measured independently and are sufficient to describe essential physical quantities.
- **Derived** dimensions/units are those that can be developed in terms of the fundamental dimensions/units.

Tables 1.1 and 1.2 list both basic, derived, and alternative units in the SI and AE systems. Figure 1.1 illustrates the relation between the basic dimensions and some of the derived dimensions. For example, squaring length results in area, cubing length results in volume, and dividing volume by time gives the volumetric flow rate. What are the dimensions of the mass flux (mass flow rate per unit area)? Can you add the appropriate lines in Figure 1.1?

The distinction between uppercase and lowercase letters should be followed even if the symbol appears in applications where the other lettering is in uppercase style. Unit abbreviations have the same form for both the singular and plural, and they are not followed by a period (except in the case of inches). One of the best fea-

TABLE 1.1 SI Units Encountered in This Book

Physical Quantity	Name of Unit	Symbol for Unit*	Definition of Unit
	Basic SI Units		
Length	metre, meter	m	
Mass	kilogramme, kilogram	kg	
Time	second	s	
Temperature	kelvin	K	
Molar amount	mole	mol	
	Derived SI Units		
Energy	joule	J	$kg \cdot m^2 \cdot s^{-2} \rightarrow Pa \cdot m^3$
Force	newton	N	$kg \cdot m \cdot s^{-2} \rightarrow J \cdot m^{-1}$
Power	watt	W	$kg \cdot m^2 \cdot s^{-3} \rightarrow J \cdot s^{-1}$
Density	kilogram per cubic meter		$kg \cdot m^{-3}$
Velocity	meter per second		$m \cdot s^{-1}$
Acceleration	meter per second squared		$m \cdot s^{-2}$
Pressure	newton per square meter, pascal		$N \cdot m^{-2}$, Pa
Heat capacity	joule per (kilogram · kelvin)		$J \cdot kg^{-1} \cdot K^{-1}$
	Alternative Units		
Time	minute, hour, day, year	min, h, d, y	
Temperature	degree Celsius	°C	
Volume	litre, liter (dm^3)	L	
Mass	tonne, ton (Mg), gram	t, g	

* Symbols for units do not take a plural form, but plural forms are used for the unabbreviated names. Non-SI units such as day (d), liter or litre (L), and ton or tonne (t) are legally recognized for use with SI.

tures of the SI system is that (except for time) units and their multiples and submultiples are related by standard factors designated by the prefix indicated in Table 1.3.

When a compound unit is formed by multiplication of two or more other units, its symbol consists of the symbols for the separate units joined by a centered dot (e.g., $N \cdot m$ for newton meter). The dot may be omitted in the case of familiar units such as watt-hour (symbol Wh) if no confusion will result, or if the symbols are separated by exponents, as in $N \cdot m^2 kg^{-2}$. Hyphens should not be used in symbols for compound units. Positive and negative exponents may be used with the symbols for the separate units either separated by a solidus or multiplied by using negative powers (e.g., m/s or $m \cdot s^{-1}$ for meters per second). **However, we do not use the center dot for multiplication in this text**. A dot can easily get confused with a period or

TABLE 1.2 American Engineering (AE) System Units Encountered in This Book

Physical Quantity	Name of Unit	Symbol
	Some Basic Units	
Length	foot	ft
Mass	pound (mass)	lb_m
Time	second, minute, hour, day	s, min, h (hr), day
Temperature	degree Rankine or degree Fahrenheit	°R or °F
Molar amount	pound mole	lb mol
	Derived Units	
Force	pound (force)	lb_f
Energy	British thermal unit, foot pound (force)	Btu, (ft)(lb_f)
Power	horsepower	hp
Density	pound (mass) per cubic foot	lb_m/ft^3
Velocity	feet per second	ft/s
Acceleration	feet per second squared	ft/s^2
Pressure	pound (force) per square inch	$lb_f/in.^2$, psi
Heat capacity	Btu per pound (mass) per degree F	Btu/(lb_m)(°F)

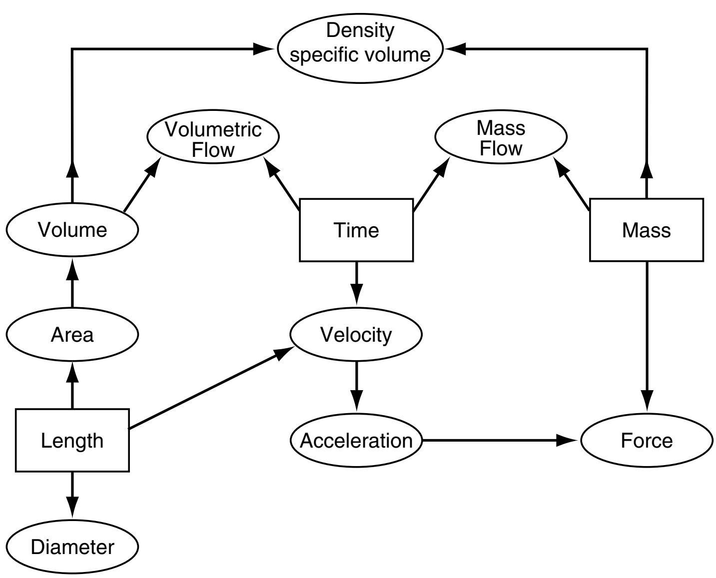


Figure 1.1 Relation between the basic dimensions (in boxes) and various derived dimensions (in ellipses).

TABLE 1.3 SI Prefixes

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^9	giga	G	10^{-1}	deci	d
10^6	mega	M	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^2	hecto	h	10^{-6}	micro	μ
10^1	deka	da	10^{-9}	nano	n

missed entirely in handwritten calculations. Instead, we will use parentheses or vertical rules, whichever is more convenient, for multiplication and division. Also, the SI convention of leaving a space between groups of numbers such as 12 650 instead of inserting a comma, as in 12,650, will be ignored to avoid confusion in handwritten numbers.

Frequently Asked Questions

1. Is the SI system of units the same as the metric system? The answer is no. SI differs from versions of the metric system (such as CGS) in the number of basic units and in the way the basic units are defined.
2. What is the major difference between the AE and USCS systems? In the USCS system the pound force is a basic unit and the pound mass a derived unit.
3. What does ms mean: millisecond or meter seconds? Mind your use of meters! The letters ms mean millisecond; the combination (m) (s) or m · s would mean meter seconds. Similarly, 1 Mm is not 1 mm! Notation such as cm^2, meaning square centimeters, frequently has to be written as $(cm)^2$ to avoid confusion.

SELF-ASSESSMENT TEST

(Answers to the self-assessment tests are listed in Appendix A.)

Questions

1. Which of the following best represents the force needed to lift a heavy suitcase:
   a. 25 N
   b. 25 kN
   c. 250 N
   d. 250 kN?
2. Pick the correct answer(s); a watt is
   a. one joule per second
   b. equal to 1 $(kg)(m^2)/s^2$

c. the unit for all types of power
d. all of the above
e. none of the above

3. Is kg/s a basic or derived unit in SI?

4. In the *IEEE Spectrum* (Jan. 2001, pp. 14–16) an article on building out the wireless Internet proposed a cell for each 0.05 km^2. Does this seem reasonable?

Problems

1. Prepare a table in which the rows are: length, volume, mass, and time. Make two columns, one for the SI and the other for the AE system of units. Fill in each row with the respective name of the unit, and in a third column show the numerical equivalency (i.e., 1 ft = 0.3048 m).

2. Classify the following units as correct or incorrect units in the SI system:
a. nm
b. °K
c. sec
d. N/mm
e. kJ/(s)(m^3)

Thought Problem

1. What volume of material will a barrel hold?

Discussion Problem

1. In a letter to the editor, the letter writer says:

> I believe SI notation might be improved so as to make it mathematically more useful by setting SI-sanctioned prefixes in **boldface** type. Then one would write, **1 c = 10 m** without any ambiguity **[c = $\mathbf{10^{-2}}$, m = $\mathbf{10^{-3}}$]** and the meaning of "mm" would be at once clear to any mathematically literate, if scientifically illiterate, citizen, namely either **$\mathbf{10^{-3}}$** m [**m**m], **$\mathbf{10^{-6}}$** [**mm**], or (after Gauss and early algebraists) m^2 [mm].
>
> With respect to the "mm" problem and remarks regarding the difference between "one square millimeter" [(**m**m)^2] and "one mili squaremeter" [**m**(m^2)], these difficulties are analogous to the confusion between a "camel's-hair brush" and a camel's hair-brush."

What do you think of the author's proposal?

1.2 Operations with Units

Answers to a question such as: how much is 2 + 2 can sometimes be debatable. You might state 4. A bad calculator might show 3.99999. What about 9 + 5? Can the answer for 9 + 5 = 2 possibly be correct? Hint: Look at a wall clock.

Every freshman knows that what you get from adding apples to oranges is fruit salad! The rules for handling units are essentially quite simple:

1.2-1 Addition, Subtraction, Equality

You can add, subtract, or equate numerical quantities only if the associated units of the quantities are the same. Thus, the operation

$$5 \text{ kilograms} + 3 \text{ joules}$$

cannot be carried out because the units as well as the dimensions of the two terms are different. The numerical operation

$$10 \text{ pounds} + 5 \text{ grams}$$

can be performed (because the dimensions are the same, mass) *only* after the units are transformed to be the same, either pounds, grams, or ounces, or some other mass unit.

1.2-2 Multiplication and Division

You can multiply or divide unlike units at will such as

$$50(\text{kg})(\text{m})/(\text{s})$$

but you cannot cancel or merge units unless they are identical. Thus, 3 m^2/60 cm can be converted to 3 m^2/0.6 m, and then to 5 m, but in m/s^2, the units cannot be cancelled or combined. In summary, units contain a significant amount of information that cannot be ignored. They also serve as guides in efficient problem solving, as you will see shortly.

Frequently Asked Question

How should you handle mathematical operations or units such as sine, log, or exponential? To be specific, if you take the log of 16 m^2 and treat the number and units as a product, then you would have

$$\log(16\text{m}^2) = \log(16) + 2\log(\text{m})$$

Various awkward ways and tricks of handling quantities such as 2 log (m) have been proposed (see, e.g., M. Karr and D. B. Loveman, "Incorporation of Units into Programming Languages," *Comma. ACM*, **21**, 385–391 [1978]). We prefer for simplicity to require that a variable be transformed or scaled to be dimensionless before you apply nonlinear operations such as log. For example, for a pipe of radius R with units of m, we would develop a dimensionless variable r^*, a fraction, for a distance r from the axis also in m, to operate on

$$r^* = \frac{r\text{m}}{R\text{m}}$$

so that

$$\log r^* = \log r + \log m - \log R - \log m = \log r - \log R = \log \frac{r}{R}$$

Can you suggest what the scaling could be for a square duct? What if the units of r are not in meters?

EXAMPLE 1.1 Dimensions and Units

Add the following:

(a) 1 foot + 3 seconds
(b) 1 horsepower + 300 watts

Solution

The operation indicated by

$$1 \text{ ft} + 3 \text{ s}$$

has no meaning since the dimensions of the two terms are not the same. One foot has the dimensions of length, whereas 3 seconds has the dimensions of time. In the case of

$$1 \text{ hp} + 300 \text{ watts}$$

the dimensions are the same (energy per unit time), but the units are different. You must transform the two quantities into like units, such as horsepower or watts, before the addition can be carried out. Since 1 hp = 746 watts,

$$746 \text{ watts} + 300 \text{ watts} = 1046 \text{ watts}$$

SELF-ASSESSMENT TEST

Questions

1. Answer the following questions yes or no. Can you
a. divide ft by s?
b. divide m by cm?
c. multiply ft by s?

d. divide ft by cm?
e. divide m by (deg) K?
f. add ft and s?
g. subtract m and (deg) K
h. add cm and ft?
i. add cm and m^2?
j. add 1 and 2 cm?

2. Why is it not possible to add 1 ft and 1 ft^2?
3. Explain how to accommodate operations such as exp and ln on a number accompanied by units.

Problems

1. Add 1 cm and 1 m.
2. Subtract 3 ft from 4 yards.
3. Divide 3 $m^{1.5}$ by 2 $m^{0.5}$.
4. Multiply 2 ft by 4 lb.

Discussion Problem

1. There seems to be two schools of thought concerning how to take the logarithm of a number that has associated dimensions. The proponents of the first school hold that taking the logarithm of a dimensioned variable is a perfectly acceptable procedure, one that leads to a dimensionless result regardless of the dimensions of the original variable. The opposing school is that taking the logarithm of a dimensioned variable is improper, and even meaningless, and the variable should be in dimensionless form before the logarithm is taken. What side do you believe is correct? Explain the reasons for your choice.

1.3 Conversion of Units and Conversion Factors

Mistakes are the usual bridge between inexperience and wisdom.

Phyllis Theroux, *Night Lights*

Columbus had many of the qualities that would appeal to today's venture capitalists. He was an experienced seafarer, prepared detailed written proposals for his ventures, and was dedicated and sincere. King John of Portugal, who rejected his first proposal in 1484, regarded him as boastful, fanciful, and overimaginative. His Portuguese experts believed that the distance to the Indies was 10,000 (U.S.) miles, four times Columbus's estimate of 2,500 (U.S.) miles. Both the experts and Columbus knew he had to travel about 68° of longitude, but Columbus apparently interpreted the Arabic literature in which the measure for 1° was 56 2/3 miles (U.S.) as ancient Italian miles, which are equal to modern 37 U.S. miles. Consequently, he thought that 68° was about 2,500 U.S. miles, whereas the correct distance was about 3900 U.S. miles.

As another example of a serious conversion error, in 1999 the Mars Climate Orbiter was lost because engineers failed to make a simple conversion from English units to SI, an embarrassing lapse that sent the $125 million craft fatally close to the Martian surface.

As a prospective engineer you must be careful of handling all sorts of units, and be able to convert a given set of units to another set with ease.

As you probably already know, the procedure for converting one set of units to another is simply to multiply any number and its associated units by ratios termed *conversion factors* to arrive at the desired answer and its associated units. Conversion factors are statements of equivalent values of different units in the same system or between systems of units used in the form of ratios. You can view a pair of (correct) conversion factors as quantities that form a ratio so that multiplying a term by the ratio is essentially the same as multiplying the term by 1.

On the inside of the front cover of this book you will find tables of commonly used conversion factors. You can locate many others in handbooks and on the Internet. Some of the references to consult can be found at the end of the chapter. Memorize a few of the common ones to save time looking them up. It will take you less time to use conversion factors you know than to look up better ones. Some web sites do the conversions for you! In the physical property software on the CD in the back of this book you can insert almost any units you want in order to retrieve property values. Nevertheless, being able to make conversions by yourself is important.

In this book, to help you follow the calculations and emphasize the use of units, we frequently make use of a special format in the calculations, as shown below. Consider the following problem:

If a plane travels at twice the speed of sound (assume that the speed of sound is 1100 ft/s), how fast is it going in miles per hour?

We formulate the conversion as follows

$$\frac{2 \times 1100 \text{ ft}}{\text{s}} \Bigg| \frac{1 \text{ mi}}{5280 \text{ ft}} \Bigg| \frac{60 \text{ s}}{1 \text{ min}} \Bigg| \frac{60 \text{ min}}{1 \text{ hr}}$$

$$\frac{\text{ft}}{\text{s}} \qquad \frac{\text{mi}}{\text{s}} \qquad \frac{\text{mi}}{\text{min}}$$

Note the format of the calculations. We have set up the calculations with vertical lines separating each ratio. These lines retain the same meaning as a ·, or parenthesis, or a multiplication sign (×) placed between each ratio. We will use this formulation frequently in this text to enable you to keep clearly in mind the significance of units in problem solving. We recommend that you always write down the units next to the associated numerical value (unless the calculation is very simple) until you become quite familiar with the use of units and can carry them in your head.

Another convenient way you can keep track of the net units in an equation is to strike through the units that can be cancelled as you proceed with the calculations. For example:

$$\left.\frac{2(1100)\cancel{\text{ft}}}{\cancel{\text{s}}}\right|\left.\frac{1\text{ mile}}{5280\ \cancel{\text{ft}}}\right|\left.\frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}}\right|\frac{60\ \cancel{\text{min}}}{1\text{ hr}}$$

At any stage in the conversion you can determine the consolidated net units and see what conversions are still required. If you want, you can do this formally, as shown above, by drawing slanted lines below the dimensional equation and writing the consolidated units on these lines; or you can do it by eye, mentally canceling and accumulating the units; or you can strike out pairs of identical units as you proceed. Consistent use of units along with numbers throughout your professional career will assist you in avoiding silly mistakes such as converting 10 centimeters to inches by multiplying by 2.54:

$$\left.\frac{10\text{ cm}}{}\right|\frac{2.54\text{ cm}}{1\text{ in.}} \neq 25.4\text{ in., instead of }\left.\frac{10\text{ cm}}{}\right|\frac{1\text{ in.}}{2.54\text{ cm}} = 3.94\text{ in.}$$

By three methods we may learn wisdom: First, by reflection, which is noblest; second, by imitation, which is easiest; and third by experience, which is the bitterest.

Confucius

Now let's look at an example.

EXAMPLE 1.2 Conversion of Units

(a) Convert 2 km to miles.
(b) Convert 400 in.^{3}/day to cm^3/min.

Solution

(a) One way to carry out the conversion is to look up a direct conversion factor, namely 1.61 km = 1 mile:

$$\left.\frac{2\text{ km}}{}\right|\frac{1\text{ mile}}{1.61\text{ km}} = 1.24\text{ mile}$$

Another way is to use conversion factors you know

$$\left.\frac{2\ \cancel{\text{km}}}{}\right|\left.\frac{10^5\ \cancel{\text{cm}}}{1\ \cancel{\text{km}}}\right|\left.\frac{1\ \cancel{\text{in.}}}{2.54\ \cancel{\text{cm}}}\right|\left.\frac{1\ \cancel{\text{ft}}}{12\ \cancel{\text{in.}}}\right|\frac{1\text{ mile}}{5280\ \cancel{\text{ft}}} = 1.24\text{ mile}$$

$$\text{(b)} \quad \frac{400\ \text{in.}^3}{\text{day}}\left|\left(\frac{2.54\ \text{cm}}{1\ \text{in.}}\right)^3\right|\frac{1\ \text{day}}{24\ \text{hr}}\left|\frac{1\ \text{hr}}{60\ \text{min}}\right. = 4.55\frac{\text{cm}^3}{\text{min}}$$

In part (b) note that not only are the numbers in the conversion of inches to centimeters raised to a power, but the units also are raised to the same power.

EXAMPLE 1.3 Nanotechnology

Nanosized materials have become the subject of intensive investigation in the last decade because of their potential use in semiconductors, drugs, protein detectors, and electron transport. Nanotechnology is the generic term that refers to the synthesis and application of such small particles. An example of a semiconductor is ZnS with a particle diameter of 1.8 nanometers. Convert this value to (a) dm (decimeters) and (b) inches.

Solution

$$\text{(a)} \quad \frac{1.8\ \text{nm}}{}\left|\frac{10^{-9}\ \text{m}}{1\ \text{nm}}\right|\frac{10\ \text{dm}}{1\ \text{m}} = 1.8 \times 10^{-8}\ \text{dm}$$

$$\text{(b)} \quad \frac{1.8\ \text{nm}}{}\left|\frac{10^{-9}\ \text{m}}{1\ \text{nm}}\right|\frac{39.37\ \text{in.}}{1\ \text{m}} = 7.09 \times 10^{-8}\ \text{in.}$$

In the AE system the conversion of terms involving pound **mass** and pound **force** deserve special attention. Let us start the discussion with Newton's Law:

$$F = Cma \tag{1.1}$$

where F = force
C = a constant whose numerical value and units depend on those selected for F, m, and a
m = mass
a = acceleration

In the SI system in which the unit of force is defined to be the Newton (N) when 1 kg is accelerated at 1 m/s^2, a conversion factor C = 1 N/(Kg)(m)/s^2 must be introduced to have the force be 1 N:

$$F = \underbrace{\frac{1\ \text{N}}{\dfrac{(\text{kg})(\text{m})}{\text{s}^2}}}_{\widetilde{C}}\left|\underbrace{\frac{1\ \text{kg}}{}}_{\widetilde{m}}\right|\underbrace{\frac{1\ \text{m}}{\text{s}^2}}_{\widetilde{a}} = 1\ \text{N} \tag{1.1}$$

Because the numerical value associated with the conversion factor is 1, the conversion factor seems simple, even nonexistent, and the units are ordinarily ignored.

In the AE system an analogous conversion factor is required. However, to make the numerical value of the force and the mass be essentially the same at the earth's surface, if a mass of 1 $1b_m$ is hypothetically accelerated at g ft/s^2, where g is the acceleration that would be caused by gravity (about 32.2 ft/s^2 depending on the location of the mass), we can make the force be 1 $1b_f$ by choosing the proper numerical value and units for the conversion factor C:

$$F = \underbrace{\left(\frac{1(\text{lb}_f)(s^2)}{32.174(\text{lb}_m)(\text{ft})}\right)}_{\widetilde{C}}\left(\underbrace{\frac{1\ \text{lb}_m}{}}_{\widetilde{m}}\middle|\underbrace{\frac{g\ \text{ft}}{s^2}}_{\widetilde{g}}\right) = 1\ \text{lb}_f \tag{1.2}$$

A numerical value of 1/32.174 has been chosen for the numerical value in the conversion factor because 32.174 is the numerical value of the average acceleration of gravity (g) (9.80665 m/s^2) at sea level at 45° latitude when g is expressed in ft/s^2. The acceleration caused by gravity, you may recall, varies by a few tenths of 1% from place to place on the surface of the earth but is quite different on the surface of the moon.

The inverse of the conversion factor with the numerical value 32.174 included is given the *special symbol* g_c

$$g_c = 32.174\frac{(\text{ft})(\text{lb}_m)}{(\text{s}^2)(\text{lb}_f)}$$

that you will see included in equations in some texts to remind you that the numerical value of the conversion factor is not a unity. To avoid confusion, we will not place g_c in the equations in this book because we will be using both SI and AE units. You will discover that the use of g_c is essential in the AE system when you need a conversion factor to adjust units when both lb_m and lb_f are involved in a calculation, or when lb_f has to be transformed to lb_m in a unit such as psia (lb_f/in.^{2}).

In summary, you can see that the AE system has the convenience that the numerical value of a pound mass is also that of a pound force if the numerical value of the ratio g/g_c is equal to 1, as it is approximately in most cases. No one gets confused by the fact that a person who is 6 feet tall has only two feet. **In this book, we will not subscript the symbol lb with m (for mass) or f (for force)** unless it becomes essential to do so to avoid confusion. We will always mean by the unit lb without a subscript the quantity pound mass. **But never forget that the pound (mass) and pound (force) are not the same units in the AE system** even though we speak of *pounds* to express force, weight, or mass.

What is the difference between mass and weight? When someone says they weigh 100 kg, or 200 pounds, how can that statement be correct when you know that **weight** is a force, not a mass, equal to the opposite of the force required to support a mass (consult some of the references at the end of this chapter for a more precise definition of weight)? To avoid confusion, just interpret the statement as follows: a person or object weighs as much as a mass of 100 kg, or 200 pounds, would weigh, if measured by a force scale.

Some Useful Trivia Concerning Conversion

A U.S. frequent-flier mile is not the same as a U.S. mile—the former is a nautical mile (1.85 km), whereas the latter is 1.61 km. In the AE system 1 m = 39.37 in., whereas for U.S. land survey applications it is 2×10^{-6} in. shorter.

EXAMPLE 1.4 A Conversion Involving Both $1b_m$ and $1b_f$

What is the potential energy in $(ft)(1b_f)$ of a 100 lb drum hanging 10 ft above the surface of the earth with reference to the surface of the earth?

Solution

The first thing to do is read the problem carefully. What are the unknown quantities? The potential energy (PE) is unknown. What are the known quantities? The mass and the height of the drum are known. How are they related? You have to look up the relation unless you recall it from physics:

$$\text{Potential energy} = P = mgh$$

Assume that the 100 lb means 100 lb mass; g = acceleration of gravity = 32.2 ft/s^2. Figure E1.4 is a sketch of the system.

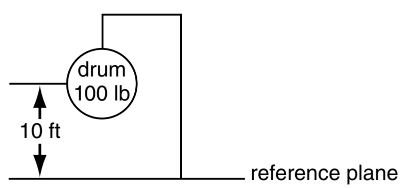


Figure E1.4

Now substitute the numerical values of the variables into the equation and perform the necessary unit conversions.

$$P = \frac{100 \text{ lb}_m}{} \left| \frac{32.2 \text{ ft}}{\text{s}^2} \right| \frac{10 \text{ ft}}{} \left| \frac{(\text{s}^2)(\text{lb}_f)}{32.174(\text{ft})(\text{lb}_m)} \right. = 1000 \text{ (ft)(lb}_f)$$

Notice that in the ratio of 32.2 ft/s^2 divided by 32.174[(ft)(lb$_m$)]/[(s^2)(lb$_f$)], the numerical values are almost equal. Many engineers would solve the problem by saying that 100 lb × 10 ft = 1000 (ft)(1b) without realizing that, in effect, they are canceling out the numbers in the g/g_c ratio, and that the lb in the solution means lb$_f$.

EXAMPLE 1.5 Conversion of Units Associated with Biological Materials

In biological systems, enzymes are used to accelerate the rates of certain biological reactions. Glucoamylase is an enzyme that aids in the conversion of starch to glucose (a sugar that cells use for energy). Experiments show that 1 μg mol of glucoamylase in a 4% starch solution results in a production rate of glucose of 0.6 μg mol/(mL)(min). Determine the production rate of glucose for this system in the units of lb mol/(ft^3)(day).

Solution

Basis: 1 min

$$\frac{0.6\ \mu\text{g mol}}{(\text{mL})(\text{min})}\left|\frac{1\text{ g mol}}{10^6\ \mu\text{g mol}}\right|\frac{1\text{ lb mol}}{454\text{ g mol}}\left|\frac{1000\text{ mL}}{1\text{ L}}\right|\frac{1\text{ L}}{3.531\times 10^{-2}\text{ ft}^3}\left|\frac{60\text{ min}}{\text{hr}}\right|\frac{24\text{ hr}}{\text{day}}$$

$$= 0.0539\frac{\text{lb mol}}{(\text{ft}^3)(\text{day})}$$

SELF-ASSESSMENT TEST

Questions

1. What is g_c?
2. Is the ratio of the numerator and denominator in a conversion factor equal to unity?
3. What is the difference, if any, between pound force and pound mass in the AE system?
4. Could a unit of force in the SI system be kilogram force?
5. Contrast the procedure for converting units within the SI system with that for the AE system.
6. What is the weight of a one pound mass at sea level? Would the mass be the same at the center of Earth? Would the weight be the same at the center of Earth?
7. What is the mass of an object that weighs 9.80 kN at sea level?

Problems

1. What are the value and units of g_c in the SI system?
2. Electronic communication via radio travels at approximately the speed of light (186,000 miles/second). The edge of the solar system is roughly at Pluto, which is 3.6×10^9 miles from Earth at its closest approach. How many hours does it take for a radio signal from Earth to reach Pluto?

3. Determine the kinetic energy of one pound of fluid moving in a pipe at the speed of 3 feet per second.
4. Convert the following from AE to SI units:
   a. 4 lb_m/ft to kg/m
   b. 1.00 $lb_m/(ft^3)(s)$ to $kg/(m^3)(s)$
5. Convert the following

$$1.57 \times 10^{-2} \text{ g/(cm)(s) to } lb_m\text{/(ft)(s)}$$

6. Convert 1.1 gal to ft^3.
7. Convert 1.1 gal to m^3.

Thought Problems

1. Comment as to what is wrong with the following statements from a textbook:
   a. Weight is the product of mass times the force of gravity.
   b. A 67-kg person on earth will weigh only 11 kg on the moon.
   c. If you have 1 g of water at 4°C that has a volume of 1.00 mL, you can use the ratio 1.00 g water/ 4°C as a conversion factor.
2. In the conversion tables in *Perry's Handbook* (5th ed.) is a row showing that the factor 0.10197 converts newtons to kilograms. Can this be correct?

Discussion Problem

1. In spite of the official adoption of the SI system of units in most countries, people still buy 10 kg of potatoes and inflate automobile tires to a value in kg (or kg/cm^2). Why does this usage occur?

1.4 Dimensional Consistency (Homogeneity)

Now that we have reviewed some background material concerning units and dimensions, we can immediately make use of this information in a very practical and important application. **A basic principle states that equations must be dimensionally consistent.** What the principle means is that each term in an equation must have the same net dimensions and units as every other term to which it is added, subtracted, or equated. Consequently, dimensional considerations can be used to help identify the dimensions and units of terms or quantities in an equation.

The concept of dimensional consistency can be illustrated by an equation that represents the pressure/volume/temperature behavior of a gas, and is known as van der Waals's equation, an equation that is discussed in more detail in Chaper 15:

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

Inspection of the equation shows that the constant a must have the units of [(pressure)(volume)^2] for the expression in the first set of parentheses to be consistent throughout. If the units of pressure are atm and those of volume are cm^3, a will have the units of [(atm)(cm)^6]. Similarly, b must have the same units as V, or in this particular case the units of cm^3. If T is in K, what must be the units of R? Check your answer by looking up R inside the front cover of the book. All equations must exhibit dimensional consistency.

EXAMPLE 1.6 Dimensional Consistency

Your handbook shows that microchip etching roughly follows the relation

$$d = 16.2 - 16.2e^{-0.021t} \quad t < 200$$

where d is the depth of the etch in microns (micrometers, μm) and t is the time of the etch in seconds. What are the units associated with the numbers 16.2 and 0.021? Convert the relation so that d becomes expressed in inches and t can be used in minutes.

Solution

After you inspect the equation that relates d as a function of t, you should be able to reach a decision about the units associated with each term on the righthand side of the equation. Both values of 16.2 must have the associated units of microns (μm). The exponential must be dimensionless so that 0.021 must have the associated units of s^{-1}. To carry out the conversion, look up suitable conversion factors inside the front cover of this book and multiply so that the units are converted from 16.2 μm to inches, and 0.021 t / s to t/min.

$$d_{\text{in}} = \frac{16.2\ \mu\text{m}}{} \left| \frac{1\ \text{m}}{10^6\ \mu\text{m}} \right| \frac{39.27\ \text{in.}}{1\ \text{m}} \left[1 - \exp \frac{-0.021}{s} \left| \frac{60s}{1\ \text{min}} \right| \frac{t_{\text{min}}}{} \right]$$

$$= 6.38 \times 10^{-4}(1 - e^{-1.26t_{\text{min}}}) \text{ inches}$$

As you proceed with the study of chemical engineering, you will find that groups of symbols may be put together, either by theory or based on experiment, that have no net units. Such collections of variables or parameters are called **dimensionless** or **nondimensional groups**. One example is the Reynolds number (group) arising in fluid mechanics.

$$\text{Reynolds number} = \frac{Dv\rho}{\mu} = N_{RE}$$

where D is the pipe diameter, say in cm; v is the fluid velocity, say in cm/s; ρ is the fluid density, say in g/cm^3; and μ is the viscosity, say in centipoise, units that can be converted to g/(cm)(s). Introducing the consistent set of units for D, v, ρ, and μ into $Dv\rho / \mu$, you will find that all the units cancel out so that the numerical value of 1 is the result of the cancellation of the units.

$$\frac{\cancel{\text{cm}}}{}\left|\frac{\cancel{\text{cm}}}{\cancel{\text{s}}}\right|\frac{\cancel{\text{g}}}{\cancel{\text{cm}^3}}\left|\frac{(\cancel{\text{cm}})\,(\cancel{\text{s}})}{\cancel{\text{g}}}\right.$$

EXAMPLE 1.7 Interesting Example of Dimensional Consistency

Explain without differentiating why the following differentiation cannot be correct:

$$\frac{d}{dx}\sqrt{1 + (x^2/a^2)} = \frac{2ax}{\sqrt{1 + (x^2/a^2)}}$$

where x *is* length and a is a constant.

Solution

Observe that x and a must have the same units because the ratio $\frac{x^2}{a^2}$ must be dimensionless (because 1 is dimensionless).

Thus, the lefthand side of the equation has units of $\frac{1}{x}$ (from d/dx). However, the righthand side of the equation has units of x^2 (the product of ax).

Consequently, something is wrong as the equation is not dimensionally consistent.

SELF-ASSESSMENT TEST

Questions

1. Explain what dimensional consistency means in an equation.
2. Explain why the so-called dimensionless group has no net dimensions.
3. If you divide all of a series of terms in an equation by one of the terms, will the resulting series of terms be dimensionless?
4. How might you make the following variables dimensionless:
   a. Length (of a pipe).
   b. Time (to empty a tank full of water).

Problems

1. An orifice meter is used to measure the rate of flow of a fluid in pipes. The flow rate is related to the pressure drop by the following equation

$$u = c\sqrt{\frac{\Delta P}{\rho}}$$

where u = fluid velocity
Δp = pressure drop (force per unit area)
ρ = density of the flowing fluid
c = constant

What are the units of c in the SI system of units?

2. The thermal conductivity k of a liquid metal is predicted via the empirical equation

$$k = A \exp\,(B/T)$$

where k is in J/(s)(m)(K) and A and B are constants. What are the units of A and B?

Thought Problems

1. Can you prove the accuracy of an equation by checking it for dimensional consistency?

2. Suppose that some short time after the "Big Bang" the laws of nature turned out to be different than the laws currently used. In particular, instead of $pV = nRT$, a different gas law arose, namely $pVT = nR$. What comments do you have about such an equation?

Discussion Problem

1. In a letter criticizing an author's equation, the writer said:

> The equation for kinetic energy of the fluid
> is not dimensionally consistent. I suggest the modification
>
> $$\text{KE} = \text{mv}^2 / 2g_c$$
>
> in which g_c is introduced. Then the units in the equation will not be $(\text{ft/s})^2$, which are the wrong units for energy.

What do you think of the comment in the letter?

1.5 Significant Figures

Decimals have a point.

Unknown

You have probably heard the story about the Egyptian tour guide who told the visitors that the pyramid they beheld in awe was 5013 years old. "Five thousand and thirteen said a visitor!" "How do you know?" "Well, said the guide, when I first began working here 13 years ago, I was told the pyramid was 5000 years old."

What do you believe about the accuracy of a statement in a travel brochure in which you read that a mountain on a trip is 8000 m (26,246 ft high)?

Responsible physical scientists and engineers agree that a measurement should include three pieces of information:

a. the magnitude of the variable being measured
b. its units
c. an estimate of its uncertainty

The last is likely to be either disassociated from the first two or ignored completely. If you have no idea of the accuracy of a measurement or a number, a conservative approach is to imply that the last digit is known within upper and lower bounds. For example, 1.43 indicates a value of 1.43 $\pm$ 0.005, meaning that the value can be deemed to be between 1.425 and 1.435. Another interpretation of 1.43 is that it means 1.43 $\pm$ 0.01.

What should you do when you add, subtract, multiply, and divide numbers that have associated uncertainty?

The accuracy you need for the results of a calculation depends on the proposed application of the results. The question is: How close is close enough? For example, in income tax forms you do not need to include cents, whereas in a bank statement cents (two decimals) are included. In engineering calculations, if the cost of inaccuracy is great (failure, fire, downtime, etc.), knowledge of the uncertainty in the calculated variables is vital. On the other hand, in determining how much fertilizer to put on your lawn in the summer, being off by 10 to 20 pounds out of 100 lb is not important.

Several options (besides common sense) exist in establishing the degree of certainty in a number. Three common decision criteria are: (1) absolute error, (2) relative error, and (3) statistical analysis.

1. First, consider the **absolute error** in a number. You have to consider two cases:

a. numbers with a decimal point, and
b. numbers without a decimal point.

For case (a), suppose we assume that the last significant figure in a number represents the associated uncertainty. Thus, the number 100.3 carries along the implication of 100.3 $\pm$ 0.05, meaning 100.3 lies in the interval between 100.25 to 100.35. Thus, 100.3 would have what is termed four significant figures. For case (a), if the number is 100.300, we will presume that additional significant figures of accuracy exist so that 100.300 will have six significant figures. (Be aware that some textbooks and authors do not attribute significance to the trailing zeros on the righthand side of a decimal point.) The rationale behind attributing additional significant figures to the trailing zeros is that they would not be added to 100.3 unless there was a reason for displaying additional accuracy. As an example, rounding the number 100.2997 to retain only six significant figures would give 100.300.

For case (b), if a number is stated without a decimal point, such as 201,300, we will assume that the trailing zeros (after the 3) do not imply any additional accuracy beyond four significant figures.

When you multiply or divide numbers, generally you should retain in your final answer the lowest number of significant figures that occur among all of the numbers involved in the calculations even though you carry along 10 or 20 digits during the calculations themselves. For example, we will treat the product (1.47)(3.0926) = 4.54612 as having only three significant figures because 1.47 has only three significant figures. The answer should be truncated to 4.55 to avoid suggesting any greater precision in the result of the multiplication.

When you add or subtract numbers, generally you should retain in your final answer the number of significant digits as determined by the error interval of the largest of the numbers. For example, in the addition

$$\begin{array}{r} 110.3 \\ +\ \underline{0.038} \\ 110.338 \end{array}$$

common sense would say to state the answer as 110.3. You should not have more than four significant figures in the sum. This decision reflects what is revealed by a more detailed examination of the error bounds imputed to the two numbers:

$$\begin{array}{rr} \text{Upper Bound} & \text{Lower Bound} \\ 110.3 + 0.05 = 110.35 & 110.3 - 0.05 = 110.25 \\ 0.038 + 0.0005 = \underline{\ \ 0.0385} & 0.038 - 0.005 = \underline{\ \ 0.0375} \\ 110.3885 & 110.2875 \end{array}$$

The midpoint of these two numbers is 110.338.

Absolute errors are easy to track and compute, but they can lead to gross distortions in the specified uncertainty of a number. For example, let's divide 98 by 93.01. You can get

$$\begin{aligned} \frac{98}{93.01} &= 1.1 \\ &= 1.05 \\ &= 1.054 \\ &= 1.0537 \end{aligned}$$

What do you think about applying the rule that states that the number of significant digits in the least precise number (two significant digits here because 98 has two significant figures) should be the number of significant digits retained in the answer? If you apply this rule, the calculated answer is 1.1, clearly a distortion of the true error in the numbers because 98 ± 1 has an error of only about 1%, whereas the result, 1.1 ± 0.1, has an error of about 10%! Certainly 1.0537 indicates too great a precision so that the choice should either be 1.05 or 1.054. Which do you think is better?

2. Perhaps the use of **relative error** can often be a better way to decide how many significant figures to retain in your answers. Suppose you divide one number by another number close to it such as 1.01/1.09 = 0.9266, and select 0.927 as the answer. The uncertainty in the answer based on the absolute error analysis is 0.001/0.927, or about 0.1%, whereas (0.01/1.09)100, or about a 1% uncertainty, existed in the original numbers. Should the relative uncertainty of the answer be fixed at about 1%, that is, truncate the answer to 0.93 rather than 0.927? Such would be the case if you applied the concept of relative error. The decision is up to you. In any case, avoid increasing the precision of your answer very much over the precision in your measurements or data when presenting results of calculations. You do have to use some common sense in applying the concept of relative error to scales that use both relative and absolute units. For example, suppose the measured error in a temperature of 25°C is 1°C, or 4%. Can you reduce the error by changing the temperature to kelvin, so that the error becomes (1/298)100 = 0.33%? Of course not.

3. A more rigorous and more complicated third way to treat uncertainty in numbers is to apply statistics in the calculations. What is involved is the concept of confidence limits for the starting numbers in a calculation, and the propagation of errors step by step through each stage of the calculations to the final result. But even a statistical analysis is not exact because we deal with nonlinear ratios of numbers. Refer to a book on statistics for further information about this approach.

In this book we base most answers on absolute error because such a choice is convenient, but will often show one or two extra figures in intermediate calculations as you should. (The numbers in your calculator are not a Holy Writ!). Keep in mind that some numbers are exact, such as the $\frac{1}{2}$ in $KE = \frac{1}{2}mv^2$ and the 2 in the superscript for the operation of square. You will also encounter integers such as 1, 2, 3, and so on, which in some cases are exact (2 reactors, 3 input streams) but in other cases are shortcut substitutes for presumed very accurate measurements in problem solving (3 moles, 10 kg).

Given a mass such as 10 kg, in which the number does not have a decimal point, despite our remarks above about trailing zeros, you can infer that quite a few significant figures apply to the mass, particularly in relation to the other values of the parameters stated in an example or problem, because you can easily measure a mass to a level of mg. You will also occasionally encounter fractions such as 2/3, which can be treated as 0.6667 in relation to the accuracy of other values in a problem. In this text for convenience we will use 273 K for the temperature equivalent to 0°C instead of 273.15 K, thus introducing an absolute error of 0.15 degrees. This is such a small error relative to the other known or presumed errors in your calculations that it can be neglected in almost all instances. Keep in mind, however, that in addition, subtraction, multiplication, and division, all of the errors that you introduce propagate into the final answer.

Feel free to round off parameters such as $\pi = 3.1416$, $\sqrt{2} = 1.414$, or Avogadro's number $N = 6.02 \times 10^{23}$. In summary, be sure to round off your answers to problems to a reasonable number of significant figures even though numbers are carried out to 10 or more digits in your computer or calculator in the intermediate calculations.

EXAMPLE 1.8 Retention of Significant Figures

If 20,100 kg is subtracted from 22,400 kg, is the answer of 2,300 kg good to four significant figures?

Solution

If you note that 22,400, 20,100, and 2,300 have no decimal points after the righthand zero, how many significant figures can you attribute input to 22,400 and 20,100? By applying the absolute error concept you can conclude that the number of significant figures is three. Scientific notation makes this decision clearer

$$\begin{array}{r} 2.24 \times 10^4 \text{ kg} \\ \underline{-2.01 \times 10^4 \text{ kg}} \\ 0.23 \times 10^4 \text{ kg} \end{array}$$

and the result retains two significant figures.

On the other hand if a decimal point were placed in each number thus, 22,400. and 20,100., indicating that the last zero was significant, then the answer of 2,300. would be valid to four significant figures.

From the viewpoint of relative error, 22,400 has an error of about 1/2% (1/224) as does 20,100 (1/201), whereas 2,300 has an error of about 5% (1/23). Should relative error have been used to establish the number of significant figures to be retained? Can you add a 0 to the right of 0.23 to give a relative error of (1/230) or about 1/2%? No. But what about giving the answer as $230. \times 10$?

EXAMPLE 1.9 Micro-dissection of DNA

A stretch-and-positioning technique on a carrier layer can be used for dissection and acquisition of a electrostatically positioned DNA strand. A device to do the micro-dissection consists of a glass substrate on which a sacrificial layer, a DNA carrier layer, and a pair of electrodes are deposited. The DNA is electrostatically stretched and immobilized onto the carrier layer with one of its molecular ends aligned on the electrode edge. A cut is made through the two layers with a stylus as a knife at an aimed portion of the DNA. By dissolving the sacrificial layer, the DNA fragment on the piece of carrier can be recovered on a membrane filter. The carrier piece can then be melted to obtain the DNA fragment in solution.

If the DNA is stretched out to a length of 48kb, and a cut made with a width of 3 μm, how many base pairs (bp) should be reported in the fragment? Note: 1 kb is 1000 base pairs (bp), and 3 kb = 1 μm.

Solution

Superficially the conversion is

$$\frac{3\ \mu\text{m}}{}\left|\frac{3\ \text{kb}}{1\ \mu\text{m}}\right|\frac{1000\ \text{bp}}{1\ \text{kb}} = 9000\ \text{bp}$$

However, because the measurement of the number of molecules in a DNA fragment can be determined to 3 or 4 significant figures in a thousand, and the 3 μm reported for the cut may well have more than 1 associated significant figure, the precision in the 9000 value may actually be better if the cut were determined to have a value of 3.0 or 3.00 μm.

SELF-ASSESSMENT TEST

Questions

1. Why can the use of absolute error in determining the number of significant digits be misleading?
2. How can you avoid a significant loss of precision in carrying out calculations involving many repetitive operations (such as addition, multiplication, and so on)?
3. Will adding a decimal point to a reported number that does not have a decimal point, such as replacing 12,600 with 12,600., improve the precision of the number?

Problems

1. Identify the number of significant figures for each of the following numbers:

3.0	23
0.353	1,000
1,000.	1,000.0

2. What is the correct sum and the number of significant digits when you add (a) 5750 and 10.3? (b) 2.000 and 0.22?
3. Convert the water flow rate of 87.0 kg of water having a density of 1000 kg/m^3 per minute to the units of gal/hr, giving the answer in the proper number of significant figures.
4. A computer chip made in Japan presumably costs \$78. The calculation to convert from yen to dollars was made as follows:

$$\left(\frac{10{,}000 \text{ yen}}{1 \text{ computer chip}}\right)\left(\frac{\$\,1.00}{128 \text{ yen}}\right) = \$\,78/\text{computer chip}$$

   Is the number of significant digits shown in the answer correct?
5. What is the answer to: $78.3 - 3.14 - 0.388$?

Thought Problems

1. Is 6 5/8 inches equivalent to (a) 51/8? (b) 6.375 inches?
2. When you want to calculate the weight of 6 silicon chips each weighing 2.35 g, is the answer good only to one significant figure, i.e., that of 6?
3. A textbook mentions the quantity of reactant as being 100 mL. How would you decide on the number of significant figures to associate with the quantity of reactant?

Discussion Problem

1. In a report of the crew laying fiber optics cable, the results for the month were listed as follows:

$$\begin{array}{r} 3000 \text{ ft} \\ 4120 \text{ ft} \\ 1300 \text{ ft} \\ \underline{2100 \text{ ft}} \\ 10{,}520 \text{ ft} \end{array}$$

   How many significant figures would you attribute to the sum?

1.6 Validation of Problem Solutions

If a mistake is not a steppingstone, it is a mistake.

Eli Siegel

Validation (sometimes referred to as verification) means checking that your problem solution is satisfactory, and possibly assessing to some extent your problem-solving procedures. By satisfactory we mean correct or close enough. Since presumably you do not know the solution before you solve the problem, trying to check your result with the unknown makes severe demands on your problem-solving skills. Unless you can compare your answer with a known one, such as the answers given in the Appendix to this and other books, what can you do? Here is a list of suggestions. (We will not consider statistical analysis.) The extent to which you can pursue a validation depends on the time you have available and the cost.

1. Repeat the calculations, possibly in a different order.
2. Start with the answer and perform the calculations in reverse order.
3. Review your assumptions and procedures. Make sure two errors do not cancel each other.
4. Compare numerical values with experimental data or data in a database (handbooks, the Internet, textbooks).
5. Examine the behavior of the calculation procedure. For example, use another starting value and check that the result changed appropriately.
6. Assess whether the answer is reasonable given what you know about the problem and its background.

The moment you have worked out an answer, start checking it–it probably isn't right.

Right Answers, *Computers and Automation,* p. 20 (September 1969)

SELF-ASSESSMENT TEST

Questions

1. Will using a calculator or computer help reduce numerical errors in your calculations?
2. What other ways of validating your answers to a problem can you suggest in addition to the one cited in Section 1.6?
3. Suppose you convert the amount of solid $CaCl_2$ in a 100 mL beaker with a net weight measured in grams to pounds, and get 2.41 lb. How would you go about checking the validity of this result?

Problems

1. Check the answer in the following calculation by starting with the answer to get the value for the original starting quantity. B is the molar density in cm^3 per gram mole of a compound, MW is the molecular weight of the compound, and ρ is the mass density of the compound in grams per cm^3.

$$\frac{\text{B}}{}\left|\frac{\rho\ \text{g mol}}{\text{cm}^3}\right|\frac{1\ \text{ft}^3}{\rho\ \text{lb}_\text{m}}\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3\left(\frac{1\ \text{m}}{35.31\ \text{ft}}\right)^3\left|\frac{1\ \text{lb}_\text{m}}{454\ \text{g}}\right|\frac{\text{MW g}}{1\ \text{g mol}} = (62.38)(\text{MW})\text{B}$$

B is the value of the variable and has the units of cm^3. Do you get B?

Looking Back

In this chapter we have reviewed the essential background you need to become skilled in converting units, applying the concept of dimensional consistency in your work, and reporting numerical values with an appropriate number of significant digits.

GLOSSARY OF NEW WORDS

Absolute error Error in a number that is a fixed value.

AE American Engineering system of units.

Conversion of units Change of units from one set to another.

Derived units Units developed in terms of the fundamental units.

Dimensional consistency Each term in an equation must have the same set of net dimensions.

Dimensionless group A collection of variables or parameters that has no net dimensions (units).

Dimensions The basic concepts of measurement such as length or time.

Force A derived unit for the product of the mass and the acceleration.

Fundamental units Units that can be measured independently.

Mass A basic dimension for the amount of material.

Nondimensional group See Dimensionless group.

Pound force The unit of force in the AE system.

Pound mass The unit of mass in the AE system.

Relative error Fraction or percent error for a number.

SI Le *Systeme Internationale d'Unites* (SI system of units).

Units Method of expressing a dimension such as ft or hour.

Validation Determination that the solution to a problem is correct.

Weight A force opposite to the force required to support a mass (usually in a gravitational field).

SUPPLEMENTARY REFERENCES

In addition to the general references listed in the FAQ in the front material, the following are pertinent:

1. Bhatt, B. I., and S. M. Vora. *Stoichiometry (SI Units)*, Tata McGraw-Hill, New Delhi (1998).

2. Horvath, A. L. *Conversion Tables in Science and Engineering*, Elsevier, New York (1986).
3. Luyben, W. L., and L. A. Wentzel. *Chemical Process Analysis: Mass and Energy Energy Balances*, Prentice-Hall, Englewood Cliffs, N. J. (1988).
4. National Institute of Standards. *The International System of Units (SI)*, NIST Special Publ. No. 330, U.S. Department of Commerce, Gaithersburg, MD 20899 (1991).
5. Reilly, P. M. "A Statistical Look at Significant Figures," *Chem. Eng. Educ.* 152–155 (Summer 1992).
6. Vatavuk, W. M. "How Significant Are Your Figures," *Chem. Eng.* 97 (August 18, 1986).

Web Sites

http://chemengineer.about.com
http://www.chemistrycoach.com/tutorials-2.html
http://www.ex.ac.uk/cimt/dictunit/dictunit.htm
http://mcgraw-hill.knovel.com/perrys
http://www.retallick.com/resources/netresrc.html
http://www.shef.ac.uk/uni/academic/A-C/cpe/mpitt/chemengs.html

PROBLEMS

(The asterisks denote the degree of difficulty, *** being the most difficult.)

***1.1** Carry out the following conversions:
(a) How many m^3 are there in $1.00(\text{mile})^3$?
(b) How many gal/min correspond to $1.00\ ft^3/s$?

***1.2** Convert
(a) $0.04\ g/(min)(m^3)$ to $lb_m/(hr)(ft^3)$.
(b) $2\ L/s$ to ft^3/day.
(c) $\dfrac{6(\text{in})(\text{cm}^2)}{(\text{yr})(\text{s})(\text{lb}_\text{m})(\text{ft}^2)}$ to all SI units.

***1.3** In a article describing an oil-shale retorting process, the authors say the retort: "could be operated at a solids mass flux well over 1,000 $lb/(h)(ft^2)$ (48k Pa/h) . . ." In several places they speak of the grade of their shale in the mixed units "34 gal (129 L)/ton." Does their report make sense?

***1.4** Convert the following:
(a) 60.0 mi/hr to ft/sec.
(b) 50.0 $lb/in.^2$ to kg/m^2.
(c) 6.20 cm/hr^2 to nm/sec^2.

***1.5** The following test will measure your SIQ. List the correct answer.

(a) Which is the correct symbol?
(1) nm (2) °K
(3) sec (4) N/mm

(b) Which is the wrong symbol?
(1) MN/m^2 (2) GHz/s
(3) $kJ/(s)(m^3)$ (4) °C/M/s

(c) Atmospheric pressure is about:
(1) 100 Pa (2) 100 kPa
(3) 10 MPa (4) 1 GPa

(d) The temperature 0°C is defined as:
(1) 273.15°K (2) Absolute zero
(3) 273.15 K (4) The freezing point of water

(e) Which height and mass are those of a petite woman?
(1) 1.50 m, 45 kg (2) 2.00 m, 95 kg
(3) 1.50 m, 75 kg (4) 1.80 m, 60 kg

(f) Which is a recommended room temperature in winter?
(1) 15°C (2) 20°C
(3) 28°C (4) 45°C

(g) The watt is:
(1) One joule per second (2) Equal to $1\ kg \cdot m^2/s^3$
(3) The unit for all types of power (4) All of the above

(h) What force may be needed to lift a heavy suitcase?
(1) 24 N (2) 250 N
(3) 25 kN (4) 250 kN

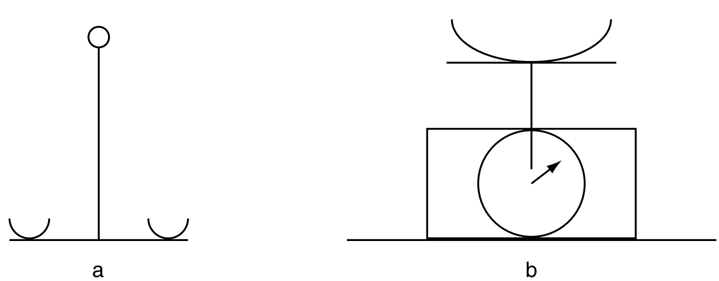
***1.6** A technical publication describes a new model 20-hp Stirling (air cycle) engine that drives a 68-kW generator. Is this possible?

****1.7** Your boss announced that the speed of the company Boeing 737 is to be cut from 525 mi/hr to 475 mi/hr to "conserve fuel," thus cutting consumption from 2200 gal/hr to 2000 gal/hr. How many gallons are saved in a 1000-mi trip?

****1.8** From Parade Magazine, August 31, 1997, page 8 by Marilyn Voss Savant:

Can you help with this problem? Suppose it takes one man 5 hours to paint a house, and it takes another man 3 hours to paint the same house. If the two men work together, how many hours would it take them? This is driving me nuts. Calculate the answer.

***1.9** Two scales are shown, a balance (a) and a spring scale (b)



In the balance calibrated weights are placed in one pan to balance the object to be weighted in the other pan. In the spring scale, the object to be weighted is placed on the pan and a spring is compressed that moves a dial on a scale in kg.

State for each device whether it directly measures mass or weight. Underline your answer. State in *one* sentence for each the reason for your answer.

***1.10** In the American Engineering system of units, the viscosity can have the units of $(\text{lb}_f)(\text{hr})/\text{ft}^2$, while in a handbook the units are (g)/(cm)(s). Convert a viscosity of 20.0 (g)/(m)(s) to the given American Engineering units.

****1.11** Thermal conductivity in the American Engineering system of units is:

$$k = \frac{\text{Btu}}{(\text{hr})(\text{ft}^2)(°\text{F/ft})}$$

Change this to:

$$\frac{\text{kJ}}{(\text{day})(\text{m}^2)(°\text{C/cm})}$$

****1.12** Water is flowing through a 2-inch diameter pipe with a velocity of 3 ft/s.

(a) What is the kinetic energy of the water in $\dfrac{(\text{ft})\,(\text{lb}_f)}{(\text{lb}_m)}$?

(b) What is the flowrate in gal/min?

***1.13** The contents of packages are often labeled in a fashion such as “net weight 250 grams.” Is it correct to so label a package?

***1.14** What is meant by a scale that shows a weight of “21.3 kg”?

***1.15** A tractor pulls a load with a force equal to 800 lb (4.0 kN) with a velocity of 300 ft/min (1.5 m/s). What is the power required using the given American Engineering system data? The SI data?

***1.16** What is the kinetic energy of a vehicle with a mass of 2300 kg moving at the rate of 10.0 ft/sec in Btu? 1 Btu = 778.2 $(\text{ft})(\text{lb}_f)$.

***1.17** A pallet of boxes weighing 10 tons is dropped from a lift truck from a height of 10 feet. The maximum velocity the pallet attains before hitting the ground is 6 ft/s. How much kinetic energy does the pallet have in $(\text{ft})(\text{lb}_f)$ at this velocity?

*****1.18** The efficiency of cell growth in a substrate in a biotechnology process was given in a report as

$$\eta = \frac{Y^c_{x/s}\,\gamma_b\,\Delta H^c_b/e^-}{\Delta H_{\text{cat}}}$$

In the notation table

η = energetic efficiency of cell metabolism (energy/energy)

$Y^c_{x/s}$ = cell yield, carbon basis (cells produced/substrate consumed)

γ_b = degree of reductance of biomass (available electron equivalents/ g mole carbon, such as 4.24 e^-equiv./mol cell carbon)

$\Delta H_b^c/e^-$ = biomass heat of combustion (energy/available electron equiv.)

ΔH_{cat}^c = available energy from catabolism (energy/mole substrate carbon)

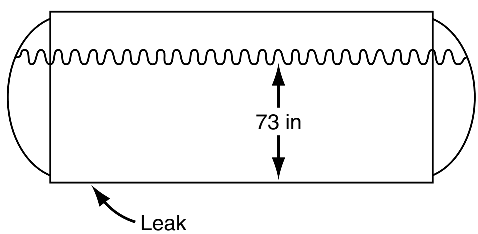
Is there a missing conversion factor? If so, what would it be? The author claims that the units in the numerator of the equation are (mol cell carbon/mol substrate carbon) (mol available e^-/mol cell carbon) (heat of combustion/mol available e^-). Is this correct?

***1.19** Leaking oil tanks have become such environmental problems that the Federal Government has implemented a number of rules to reduce the problem. A leak from a small hole in a tank can be predicted from the following relation:

$$Q = 0.61S\sqrt{(2\Delta p)/\rho}$$

where Q = the leakage rate
S = crossectional area of the leak
Δp = pressure drop
p = fluid density

To test the tank, the vapor space is pressurized with N_2 to a pressure of 23 psig. If the tank is filled with 73 inches of gasoline (sp. gr. = 0.703) and the hole is 1/4 in. in diameter, what is the value of Q (in ft^3/hr)?



****1.20** In an article on measuring flows from pipes, the author calculated q = 80.8 m^3/s using the formula

$$q = CA_1\sqrt{\frac{2gV(p_1 - p_2)}{1 - (A_1/A_2)^2}}$$

where q = volumetric flow rate, m^3/s
C = dimensionless coefficient, 0.6
A_1 = area, 2 m^2
A_2 = area, 5 m^2
V = specific volume, 10^{-3} m^3/kg
p = pressure; $p_1 - p_2$ is 50 kPa
g = acceleration of gravity

Was the calculation correct? (Answer Yes or No and explain briefly the reasoning underlying your answer.)

***1.21 The density of a certain liquid is given an equation of the following form:

$$\rho = (A + Bt)e^{CP}$$

where ρ = density in g/cm^3
t = temperature in °C
P = pressure in atm

(a) The equation is dimensionally consistent. What are the units of *A, B*, and *C*?
(b) In the units above,

$$A = 1.096$$
$$B = 0.00086$$
$$C = 0.000953$$

Find *A, B*, and *C* if ρ is expressed in lb/ft^3, t in °R, and P in lb$_f$/in.^{2}

***1.22 A relation for a dimensionless variable called the compressibility (z) is $z = 1 + \rho B + \rho^2 C + \rho^3 D$ where ρ is the density in g mol/cm^3. What are the units of B, C, and D? Convert the coefficients in the equation for z so that the density can be introduced into the equation in the units of lb$_m$/ft^3 thus: $z = 1 + \rho^* B^* + (\rho^*)^2 C^* + (\rho^*)^3 D^*$ where ρ^* is in lb$_m$/ft^3. Give the units for B*, C*, and D*, and give the equations that relate B* to B, C* to C, and D* to D.

***1.23 The velocity in a pipe in turbulent flow is expressed by the following equation

$$u = k\left[\frac{\tau}{\rho}\right]^{1/2}$$

where τ is the shear stress in N/m^2 at the pipe wall, ρ is the density of the fluid in kg/m^3, u is the velocity, and k is a coefficient. You are asked to modify the equation so that the shear stress can be introduced in the units of τ' which are lb$_f$/ft^2, and the density be ρ' for which the units are lb$_m$/ft^3 so that the velocity u' comes out in the units of ft/s. Show all calculations, and give the final equation in terms of u', τ', and ρ' so a reader will know that American Engineering units are involved in the equation.

*1.24 Without integrating, select the proper answer for

$$\int \frac{dx}{x^2 + a^2} = \begin{Bmatrix} a & \arctan & (ax) \\ a & \arctan & (x/a) \\ (1/a) & \arctan & (x/a) \\ (1/a) & \arctan & (ax) \end{Bmatrix} + \text{constant}$$

where x = length and a is a constant.

*1.25 In many plants the analytical instruments are located some distance from the equipment being monitored. Thus, some delay exists before detecting a process change and the activation of an alarm.

In a chemical plant, air samples from a process area are continuously drawn through a 1/4 in. diameter tube to an analytical instrument located 125 ft from the process area. The 1/4 in. tubing has an outside diameter of 0.25 in. (6.35 mm) and a wall thickness of 0.030 in. (0.762 mm). The sampling rate is 10 cm^3/sec under ambient conditions of 22°C and 1.0 atm. The pressure drop in the transfer line can be considered negligible. Chlorine gas is used in the process, and if it leaks from the process, it can poison workers who might be in the area of the leak. Determine the time required to detect a leak of chlorine in the process area with the equipment currently installed. You may assume the analytical equipment takes 5 sec to respond once the gas reaches the instrument. You may also assume that samples travel through the instrument sample tubing without dilution by mixing with the air ahead of the sample. Is the time excessive? How might the delay be reduced? (Adapted from Problem 13, in *Safety Health and Loss Prevention in Chemical Processes* published by the American Institute of Chemical Engineers, New York (1990).

*__1.26__ In 1916 Nusselt derived a theoretical relation for predicting the coefficient of heat transfer between a pure saturated vapor and a colder surface:

$$h = 0.943\left(\frac{k^3\rho^2 g\lambda}{L\mu\Delta T}\right)^{1/4}$$

where h = mean heat transfer coefficient, Btu/(hr) (ft^2) (Δ°F)
k = thermal conductivity, Btu/(hr) (ft)(Δ°F)
ρ = density, lb/ft^3
g = acceleration of gravity, 4.17×10^8 ft/$(hr)^2$
λ = enthalpy change, Btu/lb
L = length of tube, ft
μ = viscosity, lb_m/(hr) (ft)
ΔT = temperature difference, Δ°F

What are the units of the constant: 0.943?

*__1.27__ Explain in detail whether the following equation for flow over a rectangular weir is dimensionally consistent. (This is the modified Francis formula.)

$$q = 0.415\,(L - 0.2\,h_o)h_o^{1.5}\sqrt{2g}$$

where q = volumetric flow rate, ft^3/s
L = crest height, ft
h_o = weir head, ft
g = acceleration of gravity, 32.2 ft/$(s)^2$

*__1.28__ A useful dimensionless number called the *Reynolds number* is $\dfrac{DU\rho}{\mu}$

where D = diameter or length
U = some characteristic velocity
ρ = fluid density
μ = fluid viscosity

Calculate the Reynolds number for the following cases:

	1	2	3	4
D	2 in.	20 ft	1 ft	2 mm
U	10 ft/s	10 mi/hr	1 m/s	3 cm/s
ρ	62.4 lb/ft^3	1 lb/ft^3	12.5 kg/m^3	25 lb/ft^3
μ	0.3	0.14×10^{-4}	2×10^{-6}	1×10^{-6}
	lb_m/(hr)(ft)	lb_m/(s)(ft)	centipoise (cp)	centipoise

***1.29** Computers are used extensively in automatic plant process control systems. The computers must convert signals from devices monitoring the process, evaluate the data using the programmed engineering equations, and then feed back the appropriate control adjustments. The equations must be dimensionally consistent. Therefore, a conversion factor must be part of the equation to change the measured field variable into the proper units. Crude oil pumped from a storage unit to a tanker is to be expressed in tons/hr, but the field variables of density and the volumetric flow rate are measured in lb/ft^3 and gal/min, respectively. Determine the units and the numerical values of the factors necessary to convert the field variables to the desired output.

***1.30** If you subtract 1191 cm from 1201 cm, each number with four significant figures, does the answer of 10 cm have two or four (10.00) significant figures?

***1.31** What is the sum of

$$\begin{array}{r} 3.1472 \\ 32.05 \\ 1234 \\ 8.9426 \\ 0.0032 \\ 9.00 \end{array}$$

to the correct number of significant figures?

***1.32** Suppose you make the following sequence of measurements for the segments in laying out a compressed air line:

$$\begin{array}{r} 4.61\ \text{m} \\ 210.0\ \ \text{m} \\ 0.500\ \text{m} \end{array}$$

What should be the reported total length of the air line?

***1.33** Given that the width of a rectangular duct is 27.81 cm, and the height is 20.49 cm, what is the area of the duct with the proper number of significant figures?

***1.34** Multiply 762 by 6.3 to get 4800.60 on your calculator. How many significant figures exist in the product, and what should the rounded answer be?

***1.35** Suppose you multiply 3.84 times 0.36 to get 1.3824. Evaluate the maximum relative error in (a) each number and (b) the product. If you add the relative errors in the two numbers, is the sum the same as the relative error in their product?

***1.36** A problem was posed as follows:

The equation for the velocity of a fluid stream measured with a Pitot tube is

$$\nu = \sqrt{\frac{2\Delta p}{\rho}}$$

where ν = velocity
Δp = pressure drop
ρ = density of fluid

If the pressure drop is 15 mm Hg, and the density of the fluid is 1.20 g/cm^3, calculate the velocity in ft/s. The solution given was

$$\frac{2}{\ } \left| \frac{15\ \text{mmHg}}{\ } \right| \frac{1.013 \times 10^5\ \text{Pa}}{760\ \text{mm Hg}} \left| \frac{\frac{1\ \text{N}}{\text{m}^2}}{1\ \text{Pa}} \right| \frac{10^5(\text{g})(\text{cm})}{\underbrace{(1\ \text{N})(\text{s}^2)}_{(\text{kg})(\text{m})}} \left| \frac{(\text{cm}^3)}{1.20\ \text{g}} \right| \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2$$

$$= \left[\frac{33322.4\ \text{cm}^2}{\text{s}^2}\right]^{1/2} = 182.5\frac{\text{cm}}{\text{s}}$$

Check that the answer is correct by
(a) Repeating the calculations but carrying them out in reverse order starting with the answer.
(b) Consolidating the units and making sure the final set of units are correct.
(c) Repeating the calculations with a pressure drop of 30 mm Hg and a fluid density of 1 g/cm^3, and determining if the answer has changed in the correct proportionality.
(d) Reviewing the calculation procedure and determining if the powers have been calculated correctly and the conversion factors are correct and not inverted.

***1.37** Repeat Problem 1.36 for the solutions in (a) Example 1.2, (b) Example 1.3, (c) Example 1.4, and (d) Example 1.5.

***1.38** The dimensionless growth factor of a cell $Y^c_{X/S}$ can be represented by an input-output relation for cell growth:

$$Y^c_{X/S} = \frac{G_{ATP}\, Y_{X/ATP}}{1 + G_{ATP}\, Y_{X/ATP}}$$

where

G_{ATP} = mol ATP produced/mol carbon catabolized (utilized)
$Y_{X/ATP}$ = mole substrate carbon/mol ATP
$Y^c_{X/S}$ = dimensionless stiochiometric coefficient associated with the biomass produced in the reaction

ATP stands for the adenosine triphosphate that is involved in the catabolism.

Calculate the growth factor for the anerobic fermentation of glucose ($C_6H_{12}O_6$) to ethanol with the N supplied by NH_3 to form cells with the formula $CH_{1.75}O_{0.38}N_{0.25}$. Experiments show that $Y_{X/ATP} = 0.404$ mol cell C/mol ATP. The literature shows that 2 moles of ATP are synthesized per mole of glucose catabolized.

***1.39** Calculate the protein elongation (formation) rate per mRNA per minute based on the following data:

(a) One protein molecule is produced from x amino acid molecules.
(b) The protein (polypeptide) chain elongation rate per active ribosome uses about 1200 amino acids/min
(c) One active ribosome is equivalent to 264 ribonucleotides.
(d) $3x$ ribonucleotides equal each mRNA.

Messenger RNA (mRNA) is a copy of the information carried by a gene in DNA, and is involved in protein synthesis.